NOTATION

q, heat-flux density, W/m^2; V, rate of increase in heat-flux density, $W/m^2 \cdot sec$; V_T, heating rate, K/sec; q_m, amplitude of oscillations in heat-flux density, W/m^2; ν, modulation frequanty, Hz; τ, time, sec; T, temperature, K; T_{0e}, temperature of medium, K; T_{10}, initial temperature distribution, K; q_0, initial heat-flux density, W/m^2; x, coordinate, m; c, specific heat, J/kg·K; γ, density, kg/m^3; λ, thermal conductivity, W/m·K; ε, integral emissivity; σ, Stefan–Boltzmann constant, $W/m^2 \cdot K^4$, n, step number; δ, plate thickness, m; St, Stark number, a, thermal diffusivity; m^2/sec; $\omega = 2\pi\nu$, cyclic frequency, rad/sec; Pd, Predvoditelev number; φ, phase shift, rad.

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DYNAMICS OF THE DRAWING ZONE OF A LIGHTGUIDE BLANK FOR DIFFERENT DRAWING REGIMES WITH FURNACE AND LASER HEATING

E. M. Dianov, V. V. Kashin,
S. M. Perminov, V. N. Perminova,
S. Ya. Rusanov, and V. K. Sysoev

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The results of numerical modeling of the process of drawing a quartz blank into a lightguide with different mehtods of heating are presented. The optimal regions of the space of drawing parameters for obtaining a stable lightguide diameter are determined.

Introduction. In preparing fiber lightguides by the method of drawing from a blank (rod) many physical problems must be solved. One particular problem is to investigate the behavior of the zone of drawing between the blank and the lightguide, the so-called "onion." This question is of interest because the basic characteristics of the lightguide obtained are determined precisely by the zone of formation. In particular, the stability of the diameter along the lightguide depends on the character of the oscillation of the onion during the drawing process. For this reason much attention is devoted in the experimental and theoretical works to the behavior of the onion [1-6].

We performed a series of numerical experiments devoted to this question. The process of drawing a quartz blank into a lightguide [9, 10] was modeled by applying to this problem the methods of numerical simulation fo the motion of a viscous incompressible liquid bounded by a "free surface" [7, 8]. The motion of the quartz glass was regarded as a vertical, axisymmetric, nonstationary motion of a liquid bounded by a "free surface." All experimentally recorded situations were simulated, namely, stable continuous drawing of a blank into a lightguide, break off of the lightguide owing to capillary decomposition accompanying overheating of the drawing zone, and break off owing to underheating (viscous fracture). Thus it was established in [10] that an arbitrary combination of technological parameters is suitble for drawing. On the contrary, in the space of technological parametes of the drawing process there exist "allowed" and "forbidden" regions for the operation.

In this paper we investigate the dynamics of the behavior of the onion within the regions of parameters "allowed" for drawing for two basic methods of drawing – laser and furnace heating.

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Form of "Allowed" Regions in the Space of Drawing Parameters. To obtain the boundaries of the "allowed" regions for drawing lightguides the numerical experiment was performed as follows. The form of the drawing zone, approximated by an analytical function, was employed for the starting conditions. Within the drawing zone the temperature field, generated by heating under one or another set of conditions [9], the corresponding distribution of the viscosity [11], and the distribution of velocities, determined by the law of conservation of mass and the continuity equation, were given within the drawing zone. Then the coupled nonstationary Navier—Stokes and heat conduction equations were solved udner the condition of a moving boundary. The formulation of the problem (mathematical) and the methods of solution are described in detail in [9, 10]. The solution algorithm enabled recording the moment of breakoff of the lightguide for one or another physical reason, such as the working region losing its simply connected character. In addition, the fluctuations of the diameter of the drawn lightguide and the temperature near the point of solidification were recorded. After some time the process emerged onto a stationary state, charcterized by a stable shape of of the onion and correspnding temperature and velocity distributions of the matter in it. These data were employed as starting conditions for the second part of the numerical experiment, which will be described below. If instead of becoming stationary the drawing process stopped owing to breakoff of the lightguide, one of the technological parameters of drawing changed and the numerical experiment was repeated. In studying furnace drawing we employed the length and the temperature of the heater ℓ and T, the feed rate of the blank V_0, the drawing rate of the lightguide V, and the diameters of the blank and lightguide $2a_0$ and $2a$ as the main technological parameters. In studying laser drawing the total power of the laser radiation absorbed by the quartz glass and the width of the laser beam (P and Δ) were employed as the characteristics of the heater; the other parameters were the same. Thus it was possible to delineate the contours of the regions of parameters that are "allowed" and "forbidden" for continuous drawing of lightguides. The section of such a region by the (V, T) plane for furnace drawing is shown at the top of Fig. 1; the section by the plane (V, Δ) for laser drawing is shown in the bottom half of the figure.

Under conditions of furnace drawing the region sought is bounded from above by a zone where drawing is impossible owing to capillary instability. This effect was predicted in [7] theoretically and has been repeatedly recorded experimentally [8]. The region is bounded from below by viscous fracture of the light guide owing to underheating. Under the action of the high drawing tension the lightguide starts to deform elastically and fractures. In practice, when defects and impurities are present in the material of the blank, brittle fracture is also possible [12]. Of course, as the rate of drawing increases, generally speaking, to preserve stability the temperature of the heater must be increased at the same time.

The most characteristic feature of the laswer drawing process is the existence of a definite optimal width of the laser beam, for which a maximum drawing velocity is possible. Expansion or narrowing of the beam degrades the conditions of heating and reduces the drawing rate. The critical point (v_{max}, Δ_{opt}) is characteristic for the given values of P and $2a_0$. It, in principle, limits the possibility of obtaining a higher drawing velocity with a given radiation power and diameter of the blank.

We note that a process with small variations of the lightguide diameter (i.e., stable) cannot be obtained at all poitns of the region "allowed" for continuous drawing. The point is that in the real process of drawing lightguides there is always some instability of the technological paramters — drift of the power of the laser radiation, variations of the rate of rotation of the pulling mechanism, instability of the blank feeding unit, etc. For this reason, the zone where, firt of all, the fluctuations of the main technological parameters do not lead to cutoff of the lightguide and, second, the dynamics of the process is such that such fluctuations are accompanied by minimum oscillations of the lightguide diameter, can be regarded as optimal for practical work. Our next problem is to determine the boundaries of this zone.

Drawing of Lightguides with Furnace Heating: Response to Parameter Fluctuations. The second part of the numerical experiment was performed by the following method. As mentioned above, stationary regimes, obtained for one or another set of conditions, were employed as the starting data. At some time t the instantaneous jump-like change of one or another parameter of drawing (ΔT or ΔV) was given and the behavior of the diameter of the drawn lightguide and the temperature of the onion were recorded. In regions close to the boudnary of the "allowed" zone, even a small jump in the temperature of the heater or rate of drawing caused the lightguide to be broken off. The magnitude of this critical jump increased away from

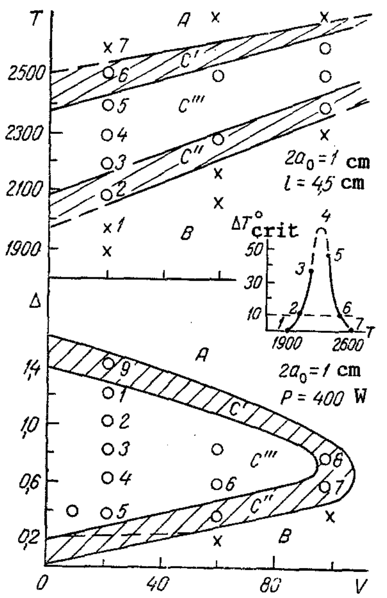

Fig. 1

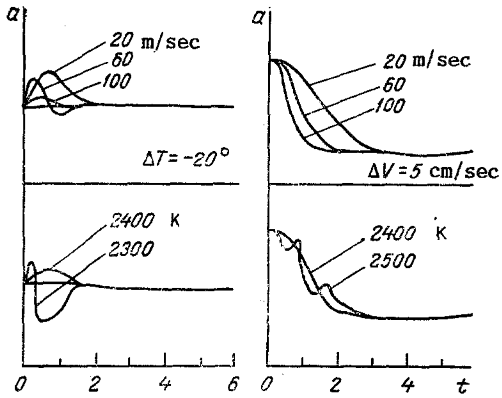

Fig. 2

Fig. 1. Diagram of the drawing regimes for furnace (top) and laser (bottom) heating. The inset shows the dependence of the critical value of ΔT on the temperature of the heater T. The space of parameters is divided into the following regions: A and B are forbidden, C' and C" are parameters that are critical with respect to external fluctuations, and C"' are parameters that are stable with respect to such fluctuations; the cross marks indicate cutoff of the lightguide and the circles indicate continuous drawing. T, K; V, cm/sec; Δ, cm.

Fig. 2. Response of variations of the lightguide diameter to the perturbing factors ΔT and ΔV for furnace drawing. a, relative units; t, sec.

the boundary. The inset in Fig. 1 shows the dependence of the critical value of ΔT_{crit} on the temperature of the heater for constant rates of feeding and drawing (i.e., along the straight line parallel to the ordinate axis in Fig. 1). One can see that this curve is really Gaussian, and in addition at the center of the region the regime can withstand without breakoff very significant changes in the temperature of the heater. Thus the region "allowed" for drawing can be conditiionally divided into critical and noncritical with respect to fluctuations of the drawing parameters, for example, with respect to the level of critical fluctuations 10°.

Next we investigated the response of the process (oscillations of the lightguide diameter) to the fluctuation ΔT = −20° and ΔV = 5 cm/sec at points in Fig. 1 marked by the numbers, i.e., along a straight line parallel to the abscissa axis (the effect of increasing the drawing rate on the process) and along a straight line parallel to the ordinate axis (the effect of increasing the heater temperature on the process). Figure 2 shows several examples of the response of the process to ΔT (on the left) and ΔV (on the right). We can draw the following conclusions.

1. When the drawing rate is increased the time during which the system emerges into a new stationary state decreases and the amplitude of the deviations of the diameter with fluctuations ΔT decreases.

2. The process as a whole is aperiodic, though under certain conditions the system can undergo several oscillations around the equilibrium position.

3. Under the action of ΔT in the critical regions the process terminates with breakoff in the remaining cases, after some time the lightguide diameter assumes the previous value. We note that for a heater temperature of 2500 K breakoff occurs owing to capillary instability – after the formation of a chain of "droplets" on the lightguide. At 2100 K these phenomena are not observed.

4. The process completes the largest number of oscillations at the center of the "allowed" region. This can be explained as follows. Two basic forces compete in the onion:

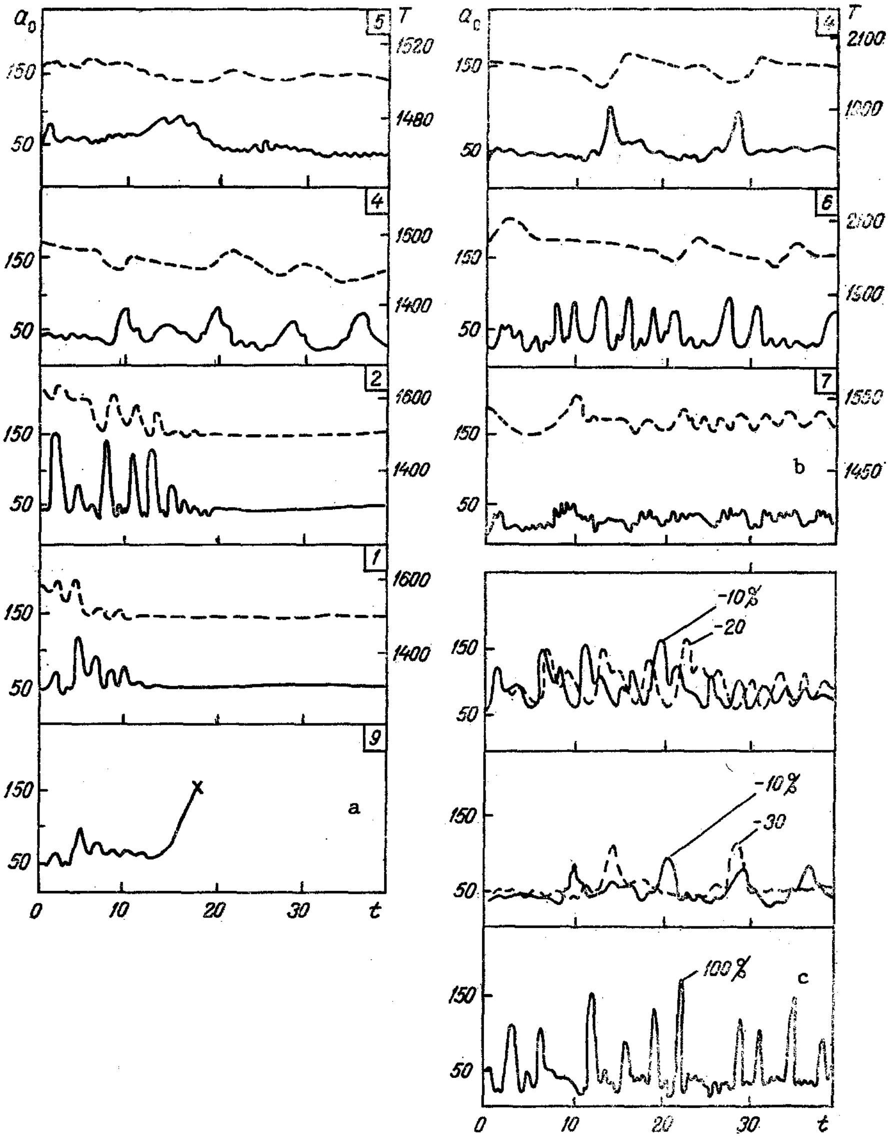

Fig. 3. Response of variations of the lightguide diameter to the perturbing factor ΔV in the case of laser heating with different laser beam width (a), drawing rates (b), and amplitudes ΔV (c). a_0, μm.

the surface tension, striving to separate the lightguide into drops and the opposing viscous friction. The surface tension force prevails at the top of the "allowed" region and the viscous friction force prevails at the bottom of the region. For this reason the process rapidly becomes stationary under the action of the force that dominates under given conditions. At the center of the zone, however, the two counteracting forces are equal in magnitude and their opposite action gives rise to several oscillations around the equilibrium position.

5. The experimental data associated with the analysis of the corresponding situations [13-15] confirm the correctness of the results obtained by numerical simulation, though they do not make it possible to study with similar completeness the lightguide drawing process.

Drawing of Lightguides with Laser Heating: Response to Fluctuations of the Parameters. The dynamic response of the onion under conditions of laser drawing at points marked by numbers in Fig. 1 was studied by an analogous method. Figure 3a shows the data describing the response of the process as a function of the value of Δ, and Fig. 3b shows the response as a fucntion of the value of ΔV (the resonse at V = 5 cm/sec is shown). The broken curves in these figures show the behavior of the temperature at the point of solidification of the lightguide. The following condlusions can be drawn from an anlaysis of these data.

1. Near the boundaries of the "allowed" zone the lightguide breaks off, but not in the same manner as it does in the case of furnace drawing. The break-off occurs after several

large oscillations of the diameter, while in the case of furnace heating there were virtually no oscillations (see Fig. 2).

2. On the whole the response of the lightguide diameter is of a more complicated character than in the case of a furnace onion. This corresponds to experimental representations of the laser onion as being more dynamic [4].

3. With a short laser onion (Δ = 0.4) high- and low-frequency (HF and LF) oscillations can be seen in the spectrum. With regard to the behavior of the temperature response (broken curves) it may be stated that thermophysical processes in the onion are associated with low frequencies, while hydrodynamic processes are associated with high frequencies.

4. For large values of Δ (long onion) the oscillations occur at one frequency, higher than LF, but decay rapidly owing to the high viscosity of quartz glass.

5. In addition, the spectrum contains oscillations associated with the strong characteristic nonlinearity of the system. The fact that the system is nonlinear follows from Fig. 3c, which shows the response to the action of ΔV with different amplitude. One can clearly see that as the magnitude of the perturbing jump ΔV increases both the amplitude of the response and its frequnecy change; this is characteristic for nonlinear systems.

6. As the drawing rate is increased the amplitude of the response decreases and its spectral composition changes at the same time – the onion becomes more stable (Fig. 3b).

This quite complicated behavior of the object under study can be qualitatively explained with the help of simple models; this will be done below.

Simple Model for Describing the Dynamics of Onion Behavior. As we have already pointed out two processes mainly affect the behavior of the system under study: "hydrodynamic oscillations" and thermal aperiodic processes [9]. These intercoupled processes can be most simply described by the following system of equations:

$$\begin{aligned} &\ddot{x} - \omega_1^2 x = 0, \\ &x = \ddot{y} + 2\delta\dot{y} + \omega_2^2 y. \end{aligned} \qquad (1)$$

Here x and y are the deviations of the diameter of the lightguide from the average value under the action of aperiodic thermal and hydrodynamic oscillation processes, respectively; ω_1 and ω_2 are the time constnat of the aperiodic process and the frequency of "hydrodynamic" oscillations, respectively; δ is the decay constant of hydrodynamic oscillations and is proportional to the viscosity of the matter in the onion. If the solution of this system is sought in the form of a harmonic oscillation, $y = e^{-i\omega t}$, then we arrive at the following quartic equation for the frequency ω of this oscilaltion:

$$\omega^4 + i\omega^3 2\delta - \omega^2(\omega_2^2 - \omega_1^2) + i\omega 2\delta\omega_1^2 - \omega_1^2\omega_2^2 = 0. \qquad (2)$$

This equation can have different solutions depending on the combination of the parameters ω_1, ω_2, and δ. As we have shown in our proceeding works, the time constant ω_1 is always small, though it increases as the length of the onion decreases and the drawing rate increase [9]. The constant ω_2 is usually quite large. The time constant δ depends strongly on the temperature (viscosity) distribution in the onion; on the whole, it is higher in long furnace onions than in short laser onions. Starting from these physical considerations, we shall analyze the solution of eq. (2).

Three basic cases are possible. For a very short laser onion (small Δ) ω_2 is quite large and exceeds ω_1. Equation (2) then has four real roots, corresponding to oscillation at two basic HF and LF frequencies (see top of Fig. 3a). For a longer laser onion the ratio of the time constants is $\delta < \omega_2$, $\omega_1 \lesssim \omega_2$. The equation has two real roots, and the oscillations occur at one fundamental frequency that is somewhat higher than LF in the first case. For a long furnace onion with strong oscillational friction the ratio $\delta >> \omega_2$, ω_1 holds. This means that Eq. (2) does not have real roots, and the behavior of the onion is aperiodic. This corresponds completely to the beiavior of the onion (see Fig. 2). In this model the nonlinearity of the system relative to the amplitude of the perturbation is neglected, but it nonetheless qualitatively explains the behavior of the oscillatory object under study.

Conclusion. Optimal Zones of the Space of Technological Lightguide-Drawing Parameters. Thus based on the data presented it may be concluded that furnace drawing of lightguides, un-

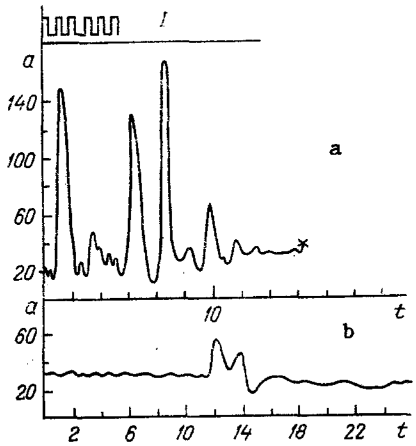

Fig. 4. Diameter of a lightguide drawn by the laser method with optimal (b) and suboptimal (a) combinations of parameters (points 5 and 2 in Fig. 1, respectively). I) shows the scheme of the modulation of the drawing velocity, simulating the instability of the drawing unit.

like laser drawing, is not critical with respect to the choice of optimal technological parameters. If the operation is conducted far from the "forbidden" regions, then almost all regimes exhibit good stability from the viewpoint of the response to fluctuations in the main technological drawing parameters. The "colder" and faster regimes are preferable.

Laser drawing, on the contrary, is very critical with respect to the choice of technological parameters. To demonstrate this we modeled the process of drawing a lightguide for different combinations of technological parameters — optimal and suboptimal — under conditions of strong instability of the pulling mechanism. The drawing rate changed periodically with a frequency of 1 Hz. The results are shown in Fig. 4. As we can see, simple optimization of the technological parameters without any control of the lightguide diameter through feedback enables significant improvement of the stability of the diameter. It should be emphasized that the parameters corresponding to a high drawing rate with a narrow laser beam are optimal.

The simplest control loops [9] enable maintaining the stability of the diameter on drawing units with laser heating at least as well as for furnace heating, preserving in the process the traditional advantages of laser heating — sterility and the possibility of creating complicated heating profiles in order to obtain strong lightguides with an elliptical cross section [16].

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ANALYSIS OF THE ACCURACY OF THE SOLUTION OF A BOUNDARY-VALUE PROBLEM ON THE BASIS OF A NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

Yu. T. Kostenko, A. S. Mazmanishvili, and I. N. Domnina

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A study is made of the accuracy of the solution of a problem concerned with the cooling of a semibounded body on the basis of the numerical inversion of the Laplace transform. The exacct and perturbed values of the image function are used as initial data.

An effective method for solving problems of heat and mass transport involves use of the Laplace transform and its subsequent inversion. The inversion problem is ill-posed (see, for example, [1, 2]) in the sense that a small change in the image-function can give rise to a large change in the original function. Often, a solution, expressed in terms of Laplace transforms, is such that it is not possible to obtain an analytical description of the result (by virtue, for example, of the transcendental nature of the expressions involved), a situation which entails the application of numerical methods for the inversion. In solving differential equations of parabolic type it is possible to apply an inversion algorithm [3], whereby one seeks the original of a function in accordance with the expression

$$F(x)=\frac{\ln 2}{x}\sum_{n=1}^{N} V_n f\left(\frac{n}{x}\ln 2\right),$$

$$V_n=(-1)^{n+N/2}\sum_{m=L}^{M}\frac{m^{N/2}(2m)!}{\left(\frac{N}{2}-m\right)!\,m!\,(m-1)!\,(n-m)!\,(2m-n)!}, \qquad (1)$$

$$L=(n+1)/2,\ M=\min(n,\ N/2).$$

The algorithm was tested on a number of tabular functions arising in heat and mass-transfer problmes; its application constituted an effective means for recovery of the original function. A comparison of the results obtained with calculations based on a known analytical representation of the function showed agreement to six significant digits. It proved convenient here to choose the number N of basis functions and, correspondingly, the number of terms in formula (1) equal to 10.

In working with systems subject to the action of random disturbances, and also in obtaining the values of functions which are the result of one or another approximating algorithm, it is important to have information concerning the noise stability of the procedure used [4]. Therefore we conducted a check on the stability of the Haver–Stefest procedure [3] used with the aid of a stochastic amplitude modulation of the image-function. Essentially, our check amounted to adding to the value of the image-function found, the latter being given analytically, a random quantity corresponding to a pattern error of the numerical algorithm. In carrying out our numerical experiments we used standard random number generators (with a uniform distribution over the interval $(-\sqrt{3},\ \sqrt{3})$ and with a normal distribution with parameters (0,

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V. I. Lenin Kharkov Polytechnical Institute, Kharkov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 3, pp. 498-501, September, 1988. Original article submitted November 20, 1986.

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